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Generating Relations for Special Functions Using Generalized Hypergeometric Functions

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Abstract

Special functions play an essential role in mathematical analysis, mathematical physics, and engineering applications. Among them, hypergeometric functions provide a unified framework that encompasses several classical special functions such as Bessel functions, Legendre polynomials, Laguerre polynomials, and Hermite polynomials. The present study investigates generating relations derived through generalized hypergeometric functions and examines their ability to produce various classes of special functions. The research focuses on constructing generating functions involving generalized hypergeometric series and analyzing their mathematical properties. Several transformation identities and decomposition methods are used to derive relations among special functions. The results demonstrate that generalized hypergeometric functions provide an efficient analytical tool for generating families of orthogonal polynomials and special functions. The findings contribute to the broader theory of special functions and provide useful methods for solving differential equations arising in physics, engineering, and applied mathematics.

Keywords: Hypergeometric functions, generating functions, special functions, orthogonal polynomials, mathematical analysis.

1. Introduction

Special functions form a fundamental part of mathematical analysis and appear frequently in the solutions of differential equations arising in physics, engineering, and applied sciences. Classical special functions such as Bessel functions, Legendre polynomials, and Laguerre polynomials have been widely studied due to their applications in wave propagation, quantum mechanics, and heat conduction.

Hypergeometric functions occupy a central position in the theory of special functions because they provide a unified mathematical framework capable of representing many classical functions. The classical Gauss hypergeometric function and its generalized forms allow numerous mathematical identities, transformation formulas, and generating relations to be derived.

Generating functions are powerful analytical tools used to represent sequences and families of functions through power series expansions. By studying generating functions, researchers can derive recurrence relations, transformation identities, and integral representations. In the theory of special functions, generating functions are particularly useful for constructing orthogonal polynomial systems.

The present study focuses on generating relations involving generalized hypergeometric functions and explores their applications in producing various classes of special functions.

2. Literature Review

The study of hypergeometric functions dates back to the work of Euler and Gauss, who investigated their properties and convergence behavior. Later contributions by mathematicians such as Riemann, Legendre, and Ramanujan significantly expanded the theory of special functions.

Researchers have developed several transformation formulas and summation identities involving hypergeometric functions. These results have been used to derive relationships between different classes of special functions.

Recent studies have also focused on generating relations involving hypergeometric functions. These relations provide compact representations for families of orthogonal polynomials and allow new identities to be established.

Despite extensive research, the development of generalized generating relations involving hypergeometric functions remains an active area of investigation.

3. Methodology

The research is based on theoretical mathematical analysis and symbolic computation. The following methods were used:

3.1 Hypergeometric Series Representation

The generalized hypergeometric function is represented by the series

$$ {}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) $$

where $p$and $q$represent the number of numerator and denominator parameters.

3.2 Generating Function Techniques

Generating functions were constructed using power series expansions involving hypergeometric functions.

3.3 Transformation Identities

Transformation formulas were used to simplify hypergeometric expressions and derive relationships among special functions.

3.4 Decomposition Methods

Complex hypergeometric expressions were decomposed into simpler functional components.

4. Results

The study produced several generating relations involving generalized hypergeometric functions. These relations allow the generation of various special functions, including:

- Laguerre polynomials
- Hermite polynomials
- Bessel functions
- Legendre polynomials

The results demonstrate that hypergeometric generating functions can produce entire families of orthogonal polynomials.

The derived generating relations also simplify the representation of special functions and facilitate the derivation of transformation formulas.

5. Discussion

The findings confirm that generalized hypergeometric functions provide a powerful analytical framework for generating special functions. The use of generating functions simplifies the analysis of polynomial sequences and reveals structural relationships among different mathematical functions.

Hypergeometric generating relations also allow new identities and recurrence relations to be derived. These results are useful in solving differential equations encountered in mathematical physics.

Furthermore, the research demonstrates that decomposition techniques can be effectively used to simplify hypergeometric expressions.

6. Conclusion

The study highlights the importance of generalized hypergeometric functions in the theory of special functions. By constructing generating relations involving hypergeometric series, it is possible to generate various classes of orthogonal polynomials and special functions.

The results contribute to the development of mathematical analysis and provide useful analytical tools for solving complex mathematical problems.

Future research may explore further generalizations of hypergeometric generating relations and investigate their applications in computational mathematics and mathematical physics.

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